2016

MATRICULATION EXAMINATION

DEPARTMENT OF MYANMAR EXAMINATION

MATHEMATICS **Time Allowed : (3) Hours**

WRITE YOUR ANSWERS IN THE ANSWER BOOKLET.

SECTION (A)

(Answer **ALL** questions. Choose the correct or the most appropriate answer for each question. Write the letter of the correct or the most appropriate answer.)

1. (1) Functions f and g are given by $f(3) = -1$ and $g(-1) = 5$. Then $(g \bullet f)^{-1}(5) =$

A. -1 B. 5 C. 3 D. 4 E. 0

(2) Given that a ⊙ b means "add 4 to a and multiply the result by b", then the value of $(2 \odot 1) \odot 3$ is

A. 55 B. 50 C. 45 D. 40 E. 30

(3) If x is a factor of $x^3 - 4x^2 + 15x + a^2 - 2a$, where a is a constant, then a =

A. 0 only B. -2 only C. 2 only D. 0 or 2 E. 0 or -2

(4) If $ax^2 + 3x - 1$ has remainder $3a + 14$ when divided by $x - 3$, then a =

A. 1 B. 2 C. 3 D. -1 E. -2

(5) The coefficient of xy^4 in the expansion of $(x - 2y)^5$ is

A. 80 B. 40 C. -10 D. -32 E. -80

(6) The middle term in the expansion of $(x + y)^{2k}$ is

A. k^{th}term B. $(k+1)^{th}$ term C. $(k + 2)^{th}$ term D. $(k + 3)^{th}$ term E.$(2k+1)^{th}$ term

(7) The solution set in R for the inequation $-4 - 3x^2 \geq 0$ is

A. $\{x \mid x \geq 0\}$ B. $\{x \mid -2 \leq x \leq \frac{2}{3}\}$ C.$\{x \mid -\frac{2}{3} \leq x \leq 2\}$ D. R E. ϕ

(8) The three angles of a triangle form an A.P. If the largest angle is twice the smallest angle, then the smallest angle is

A. 20° B. 40° C. 60° D. 80° E. 90°

(9) If $1 + 2 + 2^2 + 2^3 + ... + 2^n = 1023$, then n is

A. 9 B. 10 C. 11 D. 12 E. 18

(10) For two numbers a and b, the A.M between a and b is 5 and the G.M between a and b is 4. If $a > b$, then a^b=

A. 256 B. 64 C. 16 D. 10 E. None of these

(11) Given that $A = \begin{pmatrix} k & 3 \\ -3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$ and AB = BA, then k =

A. -4 B. 2 C. 3 D.4 E. None of these

(12) If the determinant of the matrix $\begin{pmatrix} 2x+1 & 2x \\ 3 & 2 \end{pmatrix}$ is -2, then x =

A. 2 B. 3 C. 4 D. 5 E. 6

[P. T. O

(13) The number of possible outcomes for tossing five fair coins is

A.64 B.32 C.18 D.16 E.8

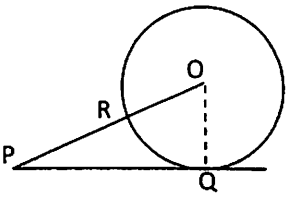
(14) Two dice are thrown 180 times. The expected frequency of obtaining total score 6 is

A.60 B.50 C.40 D.30 E.25

(15) In a cyclic quadrilateral ABCD, $\angle A = 25°$, $\angle B = 60°$. Then $\angle C - \angle D =$

A.35° B. 40° C.45° D.50° E. 55°

(16) In circle O, PQ is tangent at Q. If PQ = 4cm, the length of the diameter is 6 cm, the length of PR is



A. 6 cm B. 5 cm C. 4 cm

D. 3 cm E. 2 cm

(17) If the ratio of areas of two similar triangles is 16 : 225, then the ratio of the lengths of corresponding angle bisectors is

A. 16 : 25 B. 4 : 15 C. 25 : 4 D.6 : 25 E. 13 : 25

(18) The position vectors of A and B, relative to an origin O are $3\vec{p}+2\vec{q}, -5\vec{p}-3\vec{q}$ respectively. If M is the mid-point of AB, then the position vector of M is

A. $\vec{p}+\frac{1}{2}\vec{q}$ B. $-\vec{p}+\frac{1}{2}\vec{q}$ C. $-\vec{p}-\frac{1}{2}\vec{q}$ D. $\vec{p}-\frac{1}{2}\vec{q}$ E. $\frac{1}{2}\vec{p}+\vec{q}$

(19) Given that $\vec{a} = 3\hat{i}+4\hat{j}$. Then the vector with magnitude 20 units and in the direction of $\vec{a}$ is

A. $9\hat{i}+12\hat{j}$ B. $60\hat{i}+120\hat{j}$ C. $-12\hat{i}-16\hat{j}$ D. $12\hat{i}+16\hat{j}$ E. $21\hat{i}+28\hat{j}$

(20) If $\tan\theta = 2$, $\tan\phi = 1$, then $\cot(\theta - \phi) =$

A. –3 B. 3 C. $-\frac{1}{3}$ D. $\frac{1}{3}$ E. 1

(21) $\cos(-45°) =$

A. $\frac{2}{\sqrt{2}}$ B $\frac{-2}{\sqrt{2}}$. C. $\frac{\sqrt{2}}{2}$ D. $\frac{-\sqrt{2}}{2}$ E.1

(22) In ΔABC, BC : CA : AB = 3 : 4 : $\sqrt{37}$. Then $\angle C =$

A. 60° B. 75° C. 105° D. 120° E. 150°

(23) $\lim_{t\to\infty}(\sqrt{t^2+2t+1} - t) =$

A. 0 B. 1 C. 2 D. ∞ E. None of these

(24) The second derivative of $y = \cos x + x^2$ is

A. $-\cos x - 2$ B. $2 - \cos x$ C. $\sin x + 2x$ D.$\cos x - 2$ E. $\sin x - 2x$

(25) The coordinates of the point on the curve $y = x^3 - 6x$ at which the gradient of tangent – 3 is

A. (1, – 5) B. (–1, 5) C. (1, – 5) and (–1, 5) D. (2, –4) E. (2, 4)

(25 marks)

SECTION (B)

(Answer **ALL** questions)

2. A function $f: x \mapsto \frac{b}{x-a}, x \neq a$ and $a > 0$ is such that $(f \circ f)(x) = x$. Show that $x^2 - ax - b = 0$. (3 marks)

(OR)

Given that $x^3 - 2x^2 - 3x - 11$ and $x^3 - x^2 - 9$ have the same remainder when divided by $x + a$, determine the values of a. (3 marks)

3. If $29, a - b, a + b, 95$ is an A.P., find the values of a and b. (3 marks)

(OR)

In an G.P, the ratio of the sum of the first three terms to the sum to infinity of the G.P is 19 : 27. Find the common ratio. (3 marks)

4. A and B are two points on a circle 3 cm apart. The chord AB is produced to C making BC = 1 cm. Find the length of the tangent from C to the circle. (3 marks)

5. If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$, prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$. (3 marks)

6. Find the value of a and b for which $\frac{d}{dx}\left[\frac{\sin x}{2+\cos x}\right] = \frac{3a+b\cos x}{(2+\cos x)^2}$. (3 marks)

SECTION (C)

(Answer any **SIX** questions)

7. **(a)** Functions f and g are defined by $f: x \mapsto 2x + 1$ and $g: x \mapsto \frac{2x+5}{3-x}, x \neq 3$. Find the values of x for which $(f \circ g^{-1})(x) = x - 4$. (5 marks)

(b) Let R be the set of real numbers and a binary operation $\odot$ on R is defined by $x \odot y = xy - x - y$ for all x, y in R. Show that the operation $\odot$ is commutative. Solve the equation $(2 \odot 3) \odot x = (x \odot x) \odot 5$. (5 marks)

8. **(a)** The expression $ax^3 - (a + 3b)x^2 + 2bx + c$ is exactly divisible by $x^2 - 2x$. When the expression is divided by $x - 1$, the remainder is 8 more than when it is divided by $x + 1$. Find the values of a, b and c, hence factorize the expression completely. (5 marks)

(b) Write down and simplify the first four terms in the binomial expansion of $(1 - 2x)^7$. Use it to find the value of $(0.98)^7$, correct to four decimal places. (5 marks)

9. **(a)** Use a sketch graph to obtain the solution set of $\frac{15-4x}{4} \leq x^2$ and illustrate it on the number line. (5 marks)

(b) Find the sum of all two-digit natural numbers which are not divisible by 3.(5 marks)

[P .T. O

10.(a) Find the sum of $(b+2)+(b^2+5)+(b^3+8)+...$ to 18 terms in terms of b where $b \neq 1$. (5 marks)

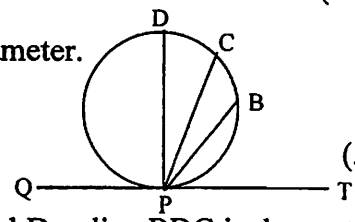
(b) Find the inverse of the matrix $M = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ and investigate whether or not the squares of M and M^{-1} are inverses of each other. (5 marks)

11.(a) Find the inverse of the matrix $\begin{pmatrix} 4 & 3 \\ 7 & 6 \end{pmatrix}$, and use it to solve the system of equations

$3y + 4x + 7 = 0$ and $14x + 12y + 32 = 0$. (5 marks)

(b) A die is rolled 360 times. Find the expected frequency of a factor of 6 and the expected frequency of a prime number. If all the scores obtained in these 360 trials are added together, what is the expected total score ? (5 marks)

12. (a) In the figure, QPT is a tangent at P and PD is a diameter. If $\angle BPT = x$, arc DC = arc CB then find $\angle DPC$, $\angle CPB$ and $\angle QPC$ in terms of x. (5 marks)



(b) Two incongruent circles P and Q intersect at A and D, a line BDC is drawn to cut the circle P at B and circle Q at C, and such that $\angle BAC = 90°$. Prove that APDQ is cyclic. (5 marks)

13.(a) A, B, C and D are four points in order on a circle O, so that AB is a diameter and $\angle COD = 90°$. If AD produced and BC produced meet at E, prove that $\alpha(\Delta ECD) = \alpha(ABCD)$. (5 marks)

(b) The coordinates of points P, Q and R are (1, 2), (7, 3) and (4, 7) respectively. If PQSR is a parallelogram, find the coordinates of S by vector method. If PS and QR meet at T, find the coordinate of T by using vectors. (5 marks)

14.(a) Given that $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{7}{5}$, prove that $\cos\alpha \cos\beta = 6 \sin\alpha \sin\beta$ and deduce a relationship between $\tan\alpha$ and $\tan\beta$. Given further that $\alpha + \beta = 45°$, calculate the value of $\tan\alpha + \tan\beta$. (5 marks)

(b) A town P is 25 miles away from the town Q in the direction N 35° E and a town R is 10 miles from Q in the direction N 42° W. Calculate the distance and bearing of P from R. (5 marks)

15.(a) Find the coordinates of the points on the curve $x^2 - y^2 = 3xy - 39$ at which the tangents are (i) parallel (ii) perpendicular to the line $x + y = 1$. (5 marks)

(b) Find the stationary points on the curve $y = 27 + 12x + 3x^2 - 2x^3$ and determine the nature of these points. (5 marks)