2015
MATRICULATION EXAMINATION
DEPARTMENT OF MYANMAR EXAMINATION

MATHEMATICS **Time Allowed: (3) Hours**

WRITE YOUR ANSWERS IN THE ANSWER BOOKLET.

SECTION (A)

(Answer **ALL** questions. Choose the correct or the most appropriate answer for each question. Write the letter of the correct or the most appropriate answer.)

1. (1) A function $f: R \rightarrow R$ is defined by $f(x) = x + 1$, then the function g such that $(g \circ f)^{-1}(x) = (x - 3)$ satisfied $g(x) =$

A. $x - 1$ B. $x - 2$ C. $x + 2$ D. $x + 1$ E. $x + 5$

(2) $\odot$ is defined on the set of real numbers by $(a - b) \odot (a + b) = ka^2 + b$. If $6 \odot 8 = 50$, then $k =$

A. 2 B. 1 C. 0 D. -1 E. -2

(3) If n is an integer, then the remainder when $2x^{2n+1} - 4x^{2n} + 5x^{2n-1} + 3$ is divided by $x+1$ is

A. -8 B. -4 C. 0 D. 4 E. 8

(4) If $x - 3$ is a factor of $x^3 - 6x^2 + ax - 6$, then $a + 4$ is

A. 22 B. 15 C. 12 D. 11 E. 5

(5) If ${}^nC_2 = 66$, then $n =$

A. 9 B. 10 C. 11 D. 12. E. 13

(6) The coefficient of the middle term in the expansion of $(x^2 + \frac{2}{x})^6$ is

A. -120 B. 125 C. 240 D. -240 E. 160

(7) The parabola $y = 12x^2 - 25x + 12$ cuts the X axis at A and B. The distance between A and B is

A. 2 B. $\frac{7}{4}$ C. $\frac{7}{12}$ D. $-\frac{7}{12}$ E. $-\frac{7}{4}$

(8) Given that 7, a, b, c, -5 in an A.P., then the mean of a, b, c is

A. -2 B. 1 C. $\frac{3}{2}$ D. 3 E. 4

(9) In a G.P. each term is positive, the 4th term is 54 and the 6th term is 486, then the common ratio is

A. 3 or -3 B. -3 only C. 3 only D. 6 only E. 6 or -6

(10) The product of the A.M. and G.M. between 4 and 16 is

A. 40 B. 60 C. 70 D. 80 E. 160

(11) The matrix $M = \begin{pmatrix} a & 4 \\ 16 & b \end{pmatrix}$ is singular and a, b are positive integers.

Then $a + b$ cannot be

A. 16 B. 20 C. 34 D. 48 E. 65

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(12) If the multiplicative inverse of the matrix $\begin{pmatrix} 1 & 3 \\ x & 1 \end{pmatrix}$ is $-\frac{1}{5}\begin{pmatrix} 1 & -3 \\ -2 & 1 \end{pmatrix}$, then x =

A. 1 B. –1 C. 3 D. 2 E. –2

(13) A letter is chosen from the letters of the word MATRICULATION. The probability that it will not be a vowel is

A. $\frac{10}{3}$ B. $\frac{7}{13}$ C. $\frac{6}{13}$ D. $\frac{4}{13}$ E. $\frac{3}{13}$

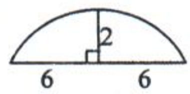
(14) Two table-tennis players P and Q played 25 games. From those games, the probability that P will win Q is 0.6. Therefore, P did not win Q in

A. 15 games B. 12 games C. 11 games D. 10 games E. 8 games

(15) The opposite angles of a cyclic quadrilateral are in the ratio 3 : 7. The difference of their degree measure is

A. 72° B. 54° C. 36° D. 18° E. none of these

(16) The arc forms part of a circle whose radius is



A. 9 B. 10 C. 16 D. 18 E. 20

(17) Two corresponding altitudes of two similar triangles are 9 cm and 12 cm. Then α(the smaller Δ) : α(the larger Δ) =

A. 4 : 5 B. 3 : 4 C. 9 : 25 D. 9 : 16 E. 16 : 25

(18) The vector of magnitude of 65 which is parallel to the vector $5\hat{i} - 12\hat{j}$ is

A. $5(5\hat{i} - 12\hat{j})$ B. $7(5\hat{i} - 12\hat{j})$ C. $13(5\hat{i} - 12\hat{j})$ D. $12\hat{i} - 5\hat{j}$ E. $6(5\hat{i} - 12\hat{j})$

(19) If $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$ and $\overrightarrow{CD} = \frac{3}{2}\overrightarrow{AB}$, then the $|\overrightarrow{CD}| =$

A. 10 B. 15 C. 25 D. 30 E. 35

(20) In any ABC, if A + B + C = 180°, then sin (A + B) =

A. – sin C B. sec(90°+ C) C. sin(90°– C) D. cos(90°– C) E. – cos C

(21) $\cos^2 60° + \sin^2 120° + 2\cot^2 135° =$

A. 4 B. 3 C. 2 D. – 2 E. – 1

(22) If θ is an acute angle and sin θ = k, then sin 2θ =

A. $2k\sqrt{1-k^2}$ B. $k\sqrt{1-k^2}$ C. $2k\sqrt{k^2-1}$ D. $k\sqrt{k^2-1}$ E. $\sqrt{k^2-1}$

(23) If $\lim_{x \to 1} \frac{a(x-1)}{x^2-1} = 2$ where a is a constant, then a =

A. 0 B. 1 C. 2 D. 3 E. 4

(24) The rate of change of the function $f : x \mapsto \frac{4}{3}x^3 - \frac{3}{4}x^2 + x - 5$ at x = 2 is

A. – 14 B. 20 C. 14 D. 12 E. 6

(25) The gradient of the tangent to the curve $y = x^2 - ax + 6$ at the point where x = 2 is –1, then the value of a is

A. – 5 B. – 3 C. 5 D. 4 E. 3

(25 marks)

SECTION (B)

(Answer **ALL** questions)

2. A function f is defined by $f(2x + 1) = x^2 - 3$. Find $a \in R$ such that $f(5) = a^2 - 8$. (3 marks)

(OR)

Given that the expression $x^3 - px^2 + qx + r$ leaves the same remainder when divided by $x + 1$ or $x - 2$. Find p in terms of q. (3 marks)

3. Given that $\sin^2 x$, $\cos^2 x$ and $5\cos^2 x - 3\sin^2 x$ are in A.P., find the value of $\sin^2 x$. (3 marks)

(OR)

Write down the next two terms of the sequence $\sqrt{2}, \sqrt{10}, 5\sqrt{2}, 5\sqrt{10}, \ldots$ and determine the n^{th} term of the sequence. (3 marks)

4. The position vectors of A, B and C are $2\vec{p} - \vec{q}, k\vec{p} + \vec{q}$ and $12\vec{p} + 4\vec{q}$ respectively. Calculate the value of k if A, B and C are collinear with $\vec{p} \neq 0, \vec{q} \neq 0, \vec{p}$ and $\vec{q}$ are not parallel. (3 marks)

5. Prove the identity $\cos 3\theta - \cos\theta = -4\sin^2\theta \cos\theta$. (3 marks)

6. Find $\lim\limits_{x \to 5} \dfrac{x^3 - 125}{5 - x}$ and $\lim\limits_{x \to \infty} \dfrac{4x^2 - 10x + 15}{2x^2 - 3x - 5}$. (3 marks)

SECTION (C)

(Answer any **SIX** questions)

7. (a) Let $f(x) = 2x - 1$, $g(x) = \dfrac{2x + 3}{x - 1}$, $x \neq 1$. Find the formula for $(g \circ f)^{-1}$ and state the domain of $(g \circ f)^{-1}$. (5 marks)

(b) Show that the mapping $\odot$ defined by $x \odot y = xy + x^2 + y^2$ is a binary operation on the set R and verify that it is commutative and but not associative. (5 marks)

8. (a) Given that $f(x) = x^3 + px^2 - 2x + 4\sqrt{3}$ has a factor $x + \sqrt{2}$, find the value of p. Show that $x - 2\sqrt{3}$ is also a factor and solve the equation $f(x) = 0$. (5 marks)

(b) Given that $(p - \frac{1}{2}x)^6 = r - 96x + sx^2 + \ldots$, find p, r, s. (5 marks)

9. (a) Find the solution set of the inequation $3x^2 < x^2 - x + 3$, by graphical method and illustrate it on the number line. (5 marks)

(b) The sum of four consecutive numbers in an A.P. is 28. The product of the second and third numbers exceeds that of the first and last numbers by 18. Find the numbers.

(5 marks)

10.(a) A geometric progression has three terms a, b, c whose sum is 42. If 6 is added to each of the first two terms and 3 to the third, a new G.P. results whose first term is the same as b. Find a, b and c. (5 marks)

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(b) Given that $A = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 2 \\ -5 & -3 \end{pmatrix}$, write down the inverse matrix A^{-1} and find the matrix P and Q such that $PA = 2I$ and $AQ = 2B$. (5 marks)

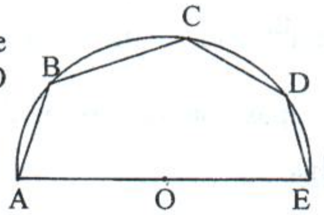
11.(a) Find the solution set of the system of equations

$2x - 5y = 1$

$3x - 7y = 2$ by matrix method. (5 marks)

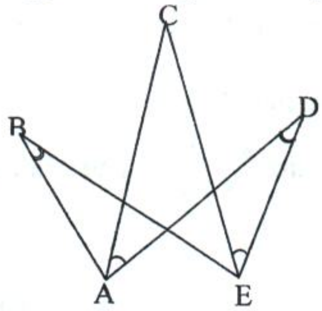
(b) The probability of an event A happening is $\frac{2}{3}$ and the probability that an event B happening is $\frac{3}{8}$. Given that A and B are independent, calculate the probability that neither event happens and just one of the two events happens. (5 marks)

12.(a) In the figure, ABCDE is a semicircle at centre O, the segment AE is the diameter and B, C, D are any points on the arc. Prove that $\angle ABC + \angle CDE = 270°$. (5 marks)



(b) Given $\angle ABE = \angle ADE$ and $\angle DAC = \angle DEC$.

Prove A, B, C, D and E all lie on one circle. (5 marks)



13.(a) ABC is a triangle. If BPC,CQA,ARB are equilateral triangles and

$\alpha(\Delta BPC) + \alpha(\Delta CQA) = \alpha(\Delta ARB)$, then prove that ABC is a right triangle. (5 marks)

(b) In a quadrilateral OLNM , OM//LN ,where $\vec{OL} = \vec{a}$, $\vec{OM} = \vec{b}$ and $\vec{LN} = k\,\vec{b}$. OP is drawn parallel to MN to meet the diagonal ML at P. If $LP = \frac{1}{4} LM$. Find the value of k. (5 marks)

14.(a) Show that $\sin(\alpha + \beta) \cdot \sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta$. (5 marks)

(b) Given that $\sin A = \frac{2}{\sqrt{5}}$, $\cos B = -\frac{\sqrt{2}}{3}$ and that both A and B are in the same quadrant, calculate the value of each of the following:

(i) $\cos(A + B)$ (ii) $\cos(2A - B)$. (5 marks)

15.(a) If $y = A\cos(\ell n\, x) + B\sin(\ell n\, x)$, where A and B are constants; show that $x^2y'' + xy' + y = 0$. (5 marks)

(b) Given that $x + y = 5$, calculate the minimum value of $x^2 + xy + y^2$. (5 marks)